

Universal post-quench prethermalization at a quantum critical point

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(Dated: March 11, 2015)

We consider an open system near a quantum critical point that is suddenly moved towards the critical point. The bath-dominated diffusive non-equilibrium dynamics after the quench is shown to follow scaling behavior, governed by a critical exponent that emerges in addition to the known equilibrium critical exponents. We determine this exponent and show that it describes universal prethermalized coarsening dynamics of the order parameter in an intermediate time regime. Implications of this quantum critical prethermalization are a powerlaw rise of order and correlations after an initial collapse of the equilibrium state and a crossover to thermalization that occurs arbitrarily late for sufficiently shallow quenches.

Predicting the out-of-equilibrium dynamics of quantum many-body systems is a challenge of fundamental and practical importance. This research area has been boosted by recent experiments in cold-atom gases [1] and scaled-up quantum-circuits [2], by ultra-fast pump-probe measurements in correlated materials [3–5], and by performing heavy-ion collisions that explore the quark-gluon plasma [6]. In this context, the universality near a quantum critical point (QCP), well established in and near equilibrium, comes with the potential to make quantitative predictions for strongly interacting systems far from equilibrium. For example, the quantum version [7–11] of the Kibble-Zurek mechanism of defect formation [12, 13] was developed for systems driven through a symmetry breaking QCP at a small, but finite rate. Similarly, near a QCP the long-time dynamics after a sudden change of Hamiltonian parameters, is governed by equilibrium exponents [14]. These phenomena occur in the regime of longest time scales.

Recently, however, many physical systems away from equilibrium were identified which display novel dynamical behavior on intermediate time scales, a behavior often referred to as prethermalization [15–25]. The question arises whether one can expect universality during prethermalization if one drives a system towards a QCP. Even if this is done at a finite rate $1/\tau$, a system will fall out of equilibrium at some point, a behavior owed to the critical slowing down near the QCP. Then a scaling theory with characteristic time scale τ can be developed [10], where regions of the size of the freeze-out length $\propto \tau^{1/z}$ emerge that behave like in equilibrium. z is the dynamic critical exponent. In case of a quantum quench, the time scale τ and the freeze-out length become comparable to microscopic time and length scales, respectively and the system instantly falls out of equilibrium. The detailed recovery of this out-of-equilibrium dynamics, along with the time dependence of length scales, order-parameter correlations, and the potential for out-of-equilibrium universality are major theoretical and experimental challenges.

In this Letter, we show that the time evolution of

observables in an open system that is suddenly moved to a QCP displays universal behavior (see Fig. 1(a-b)). Their non-equilibrium dynamics is governed by a critical exponent that describes the slow decay of post-quench correlations and response soon after a quantum-quench, where initial correlations are still important. It is therefore not related to equilibrium exponents. This behavior is astounding as universality is usually reserved for large time and length scales. From the value of the exponent we conclude that initial state correlations rapidly collapse after a quench and that the order parameter undergoes an intermediate coarsening, i.e. grows due to the growing *light-cone* length $\xi(t) \propto t^{1/z}$ (see Fig. 1(c)),

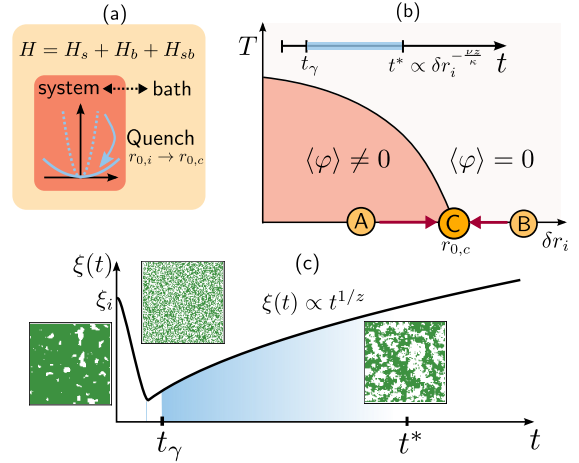


FIG. 1. (Color online) (a) Schematic description of the setup and quench protocol. (b) Schematic phase diagram as a function of temperature T and mass $\delta r_i = r_{0,i} - r_{0,c}$. Red arrows describe the quench protocol. Dynamics exhibits three time regimes: $t < t_\gamma = \gamma^{-z/2(z-1)}$ with non-universal dynamics, the universal prethermalized regime $t_\gamma < t < t^* \propto \delta r_i^{-\nu/\kappa}$ that we study, and a quasi-adiabatic regime $t > t^*$ described by equilibrium critical exponents. Here, γ is system-bath coupling and κ/ν the scaling dimension of δr_i . (c) Correlation length collapse and light-cone like revival following a quench with initial length ξ_i . Inset: sketches of order parameter configurations with domains of typical size $\xi(t)$.

before it decays quasi-adiabatically at longer times. We also demonstrate that the duration of this intermediate prethermalization can be manipulated and tuned to be arbitrarily large. While there are important differences between classical and quantum quenches, the analysis of this paper was motivated by the pioneering theory of classical dynamics in Ref. 26 (see also Ref. [27] for the case of colored noise).

The quench protocol that underlies our analysis is indicated in Fig. 1(a-b). We consider a quantum many-body system that is coupled to an external bath of harmonic oscillators. Prior to the quench, the complete system is prepared in the ground state of the initial Hamiltonian $H_i = H_{s,i} + H_b + H_{sb}$. The initial Hamiltonian of the system, $H_{s,i}$, describes a N -component scalar quantum field $\varphi(x, t)$ with components φ_a ($a = 1, \dots, N$):

$$H_{s,i} = \frac{1}{2} \int d^d x \left(\pi^2 + r_{0,i} \varphi^2 + (\nabla \varphi)^2 + \frac{u}{2} \varphi^4 \right), \quad (1)$$

where π is the canonically conjugated momentum to φ . $H_b = \frac{1}{2} \int d^d x \sum_l (\Omega_l^2 \mathbf{X}_l^2 + \mathbf{P}_l^2)$ describes the external bath of harmonic oscillators and $H_{sb} = \sum_l c_l \int d^d x \mathbf{X}_l \cdot \varphi$ the coupling between system and bath. Next, we suddenly change $H_{s,i} \rightarrow H_s$ by switching $r_{0,i} \rightarrow r_{0,c}$ to its value right at the QCP of system + bath in equilibrium (see Fig. 1(b)). The time evolution after the quench is now governed by the new Hamiltonian $H = H_s + H_b + H_{sb}$. The bath ensures that the system eventually equilibrates at $T = 0$, which allows reaching the QCP for $t \rightarrow \infty$. A crucial variable is the distance to the critical point $\delta r_i = r_{0,i} - r_{0,c}$ before the quench. After the quench we consider $\delta r_f = r_{0,f} - r_{0,c} = 0$, while the same behavior is expected for generic quenches that move the system closer to the critical point $\delta r_f \ll \delta r_i$ or take place at a finite but small temperature $T \ll \delta r_i^{\nu z} / \gamma^{z/2}$ with system-bath coupling γ defined below.

The Hamiltonian $H_{s,i}$ of Eq.(1) describes a transverse-field Ising model for $N = 1$, systems near a superconducting-insulator quantum phase transition, Josephson junction arrays and quantum antiferromagnets in an external magnetic field for $N = 2$, or quantum dimer systems for $N = 3$ [28]. Our theory for system + bath can then be applied to a range of systems such as dissipative superconducting nano-wires [29], the superfluid-insulator transition in cold-atom gases coupled to other bath-atoms [30], or low-dimensional Heisenberg spin-dimers or transverse field Ising spins with strong quantum fluctuations and coupling to phonons. Another promising realization can be achieved by an ensemble of qubits in a photon cavity [2, 31]. The effects of the bath are described in terms of $\eta(\omega) = -\sum_l \frac{c_l^2}{(\omega + i0^+)^2 - \Omega_l^2}$. We consider for the spectral density of the bath:

$$\text{Im } \eta(\omega) = \gamma \omega |\omega|^{\alpha-1} e^{-|\omega|/\omega_c} \quad (2)$$

with damping coefficient γ and cut-off energy ω_c . The exponent α determines the low-energy spectrum of the

bath, where $\alpha = 1$ corresponds ohmic damping while $\alpha > (<)1$ corresponds to super-ohmic (sub-ohmic) damping [32]. In the following, we consider the hierarchy of scales $\omega_c \gg t_\gamma^{-1} = \gamma^{1/(2-\alpha)}$ and analyze the regime $t > t_\gamma$ when the dynamics is dominated by the bath. For the one-loop RG analysis used in this paper, it holds $z = 2/\alpha$.

We start with general scaling arguments for the non-equilibrium dynamics after a quench towards the QCP. The scaling behavior will be confirmed using a perturbative renormalization group (RG) analysis later in the paper. In equilibrium, the order parameter behaves as function of the distance δr to the QCP according to $\langle \varphi_a(\delta r) \rangle_{\text{eq}} = b^{-\beta/\nu} \langle \varphi_a(b^{1/\nu} \delta r) \rangle_{\text{eq}}$, with scaling parameter $b > 1$, which leads to the well known behavior $\langle \varphi_a(\delta r) \rangle_{\text{eq}} \propto \delta r^\beta$. In our case δr rapidly changes as function of time from δr_i to δr_f , leading to a t -dependence of the order parameter. The generalization of the equilibrium scaling relation can be performed in analogy to boundary layer scaling theory as it occurs near surfaces and interfaces [33]. Here, a new healing length scale associated with surface fields appears. In our problem, the boundary layer incorporating the initial value problem corresponds to a “surface in time” [34–36] and exhibits an associated new healing time scale t^* . Following Ref. 33 it follows for the order parameter $\langle \varphi_a(\delta r_i, \delta r_f, t) \rangle = b^{-\beta/\nu} \langle \varphi_a(b^{\kappa/\nu} \delta r_i, b^{1/\nu} \delta r_f, b^{-z} t) \rangle$. While δr_f scales as in equilibrium, reflecting the fact that the system approaches equilibrium for $t \rightarrow \infty$, the initial mass δr_i has a nontrivial scaling exponent κ/ν . For $\delta r_f = 0$, i.e. a quench right to the QCP, it follows with $b = t^{1/z}$:

$$\langle \varphi_a(t, \delta r_i) \rangle = t^{-\frac{\beta}{\nu z}} \Phi(t^{\frac{\kappa}{\nu z}} \delta r_i), \quad (3)$$

with universal function $\Phi(y)$. As shown in Fig. 2, in the long time limit $t \gg t^*$ for $\Phi(y \gg 1) \rightarrow \text{const.}$, the order parameter decays quasi-adiabatically ($\langle \varphi_a(t) \rangle \propto t^{-\frac{\beta}{\nu z}} \propto \xi(t)^{-\beta/\nu}$) to zero with timescale

$$t^* \propto \delta r_i^{-\frac{\nu z}{\kappa}}. \quad (4)$$

In the opposite limit $t \ll t^*$ the situation is qualitatively different. Assuming in analogy to Ref. 33 that the susceptibility with respect to a temporal boundary-layer term $\langle \varphi_{a,i} \rangle \propto \delta r_i^\beta$ is finite, it follows $\Phi(y \ll 1) \propto y^\beta$, such that

$$\langle \varphi_a(t) \rangle \propto t^\theta \quad \text{with} \quad \theta = \frac{(\kappa - 1)\beta}{\nu z}. \quad (5)$$

Thus, a new universal time dependence of the order parameter emerges at short times. The value of the exponent θ is determined by the scaling dimension κ of δr_i . The time scale t^* separates the regime governed by the initial quench and concomitant fall out of equilibrium from the quasi-adiabatic long time behavior. Thus, in analogy to spatial boundary layer problems it describes the dynamic healing after the quench.

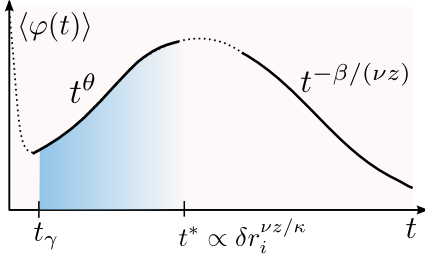


FIG. 2. (Color online) Schematic order parameter dynamics $\langle \varphi(t) \rangle$. In the prethermalized regime $t_\gamma < t < t^*$ (blue) it is governed by new universal critical exponent θ . At longer times, $\langle \varphi(t) \rangle$ decays to zero quasi-adiabatically as described by equilibrium exponents.

The same exponent θ also determines the time dependence of correlation and response functions. To analyze the non-equilibrium dynamics we employ the Keldysh formalism of many-body theory [37] and use the specific form of the Keldysh contour of Ref. [38], appropriate for our quench protocol. The key quantities are the retarded response function G^R and the Keldysh correlation function G^K :

$$\begin{aligned} G^R(k, t, t') &= -i\theta(t - t') \langle [\varphi_a(k, t), \varphi_a(-k, t')]_- \rangle \\ G^K(k, t, t') &= -i \langle [\varphi_a(k, t), \varphi_a(-k, t')]_+ \rangle \end{aligned} \quad (6)$$

with momentum k . They are no longer related by the fluctuation-dissipation theorem. We expect from dimensional arguments

$$iG^{R(K)}(k, t, t') = \left(\frac{t}{t'}\right)^{\theta(\theta')} \frac{f^{R(K)}(k^z t / \gamma^{z/2}, t'/t)}{k^{2-\eta-z\gamma^{z/2}}}. \quad (7)$$

In an out-of-equilibrium state the correlation and response functions depend on both time variables. This gives rise to an additional dimensionless ratio t/t' compared to scaling in equilibrium. The singular dependence on this ratio in G^R and G^K is characterized by exponents θ and θ' , respectively. Thus, the scaling functions f^R and f^K depend only weakly on t'/t if $t \gg t'$. The exponents θ and θ' are not independent. Relating G^R and G^K in the Dyson equation yields $\theta = \theta' + \frac{2-z-\eta}{z}$.

Let us now demonstrate that θ in Eqs. (5) and (7) is indeed the same. We consider an initial state characterized by a finite order parameter $\langle \varphi_i \rangle$ (path $A \rightarrow C$ in Fig. 1(b)). A region of volume $\xi(t)^d$ is correlated at time t after the quench and Eq. (7) yields for the local, i.e. momentum averaged, correlation function $G_{loc}^K(t, t') \propto (t/t')^{\theta'} t^{-\frac{d-\eta-z+2}{z}}$. The initial order parameter $\langle \varphi_i \rangle$ polarizes the system for a certain time. The magnetization at time t is then $\langle \varphi_i \rangle$ multiplied by the local correlation function up to t and the size of the correlation volume: $\langle \varphi(t) \rangle \simeq \langle \varphi_i \rangle iG_{loc}^K(t, t') \times t^{d/z}$. We obtain the powerlaw behavior of the order parameter of Eq.(5). The time dependence of the order parameter is

therefore a balance between the decay of local correlations encoded in $G_{loc}^K(t, t')$ and the growth of the volume encompassed by light-cone propagation, i.e. $\xi(t)$.

Next we demonstrate this behavior in an explicit analysis and determine the value of the exponent θ . We start using simple perturbation theory and perform a more rigorous renormalization group analysis in the second step. At time t after the quench correlations are limited by the light-cone. This gives rise to a time dependent mass $r(t) = \gamma a/t^{2/z}$ in the propagator, where a is a dimensionless coefficient. Scattering events caused by collisions of excitations in regions of t -dependent size turn out to be highly singular. A perturbation theory in a that includes such scattering events yields to leading order and for $t' \ll t$:

$$G^R(k, t, t') = G_0^R(k, t) [1 + \theta \log(t/t') + \dots], \quad (8)$$

where the omitted terms are non-singular for $t' \rightarrow 0$ and

$$\theta = -\frac{a \sin(\pi/z)}{\Gamma(2/z)}. \quad (9)$$

G_0^R is the bare retarded Green's function given in the supplementary section [39]. Exponentiation of the logarithm leads to Eq.(7).

We now perform a momentum-shell RG approach to sum up these logarithms in a controlled fashion and determine the exponent θ . In full analogy to the equilibrium case we integrate out states in a shell with momenta $\Lambda/b < k < \Lambda$ with $b > 1$ and rescale fields, momenta and time variables. The small parameter controlling the calculation is the deviation from the upper critical dimension $\epsilon = 4 - d - z$. The mass δr_i in the initial Hamiltonian is a strongly relevant perturbation and rapidly flows to large values. The non-equilibrium dynamics of the system is therefore governed by the deep-quench fixed point $(\hat{u}^*, \delta r_i^*, \delta r_f^*) = (\hat{u}^*, \infty, 0)$. Here $\hat{u}^* = \frac{c_z}{N+8}\epsilon$ is the equilibrium value of the dimensionless coupling constant $\hat{u} = uK_d\Lambda^{-\epsilon}/\gamma^{z/2}$ with $K_d = \frac{\Gamma(d/2)}{2\pi^{d/2}(2\pi)^d}$ and coefficient $c_z = \frac{4 \sin(\pi z/2)}{z(2-z) \sin^{z/2}(\pi/z)}$. The scaling dimension of δr_i is relative to the fixed point $\delta r_i^* = \infty$, i.e. $1/\delta r_i \propto b^{-\kappa/\nu}$ is a dangerously irrelevant variable at the deep quench fixed point.

We work with $\delta r_i > 0$ corresponding to a quench out of the unbroken phase and assume that θ is the same for the two paths $A \rightarrow C$ and $B \rightarrow C$. For the mass renormalization after the quench follows at one-loop

$$r_f'(t) = b^2 r_f(b^z t) + u \frac{N+2}{2} \int^> \frac{d^d k}{(2\pi)^d} iG_0^K(k, t, t), \quad (10)$$

where $>$ refers to momenta inside the shell. In equilibrium $r_f(t)$ is t -independent and we recover the usual one-loop result for the mass renormalization. The quench

mixes $r_f(t)$ at different times during the flow. For a similar analysis of classical surface criticality, see Ref. 40. We replace δr_i , that enters G_0^K , and \hat{u} by their deep-quench fixed-point values. From Eq. (10) we then obtain a differential equation for the corresponding time-dependent fixed-point mass $r_f^*(t)$:

$$2r_f^* + zt \frac{dr_f^*}{dt} + \frac{(N+2)\hat{u}^* \Lambda^2}{2} f_0^K(\Lambda^z t / \gamma^{z/2}, 1) = 0. \quad (11)$$

The scaling function f_0^K characterizes G_0^K according to Eq. (7). The solution of Eq. (11) is

$$r_f^*(t) = \frac{\gamma a}{t^{2/z}} - \frac{(N+2)\hat{u}^* \Lambda^2}{2zt^{2/z}} \int^t dt' f_0^K\left(\frac{\Lambda^z t'}{\gamma^{z/2}}, 1\right) t'^{\frac{2-z}{z}}, \quad (12)$$

where a denotes the integration constant of Eq. (11). We find $f_0^K(\Lambda^z t / \gamma^{z/2} \rightarrow \infty, 1) \rightarrow f_{eq,0}^K$, where $f_{eq,0}^K$ describes the equal-time Keldysh function in equilibrium after the quench. For a perturbative RG analysis a long range decay of the mass parameter cannot emerge. We can therefore fix the integration constant a from the condition that $r_f^*(t)$ rapidly approaches its equilibrium value, i.e. that $\delta r_f^*(t) = r_f^*(t) - r_{eq}^* \rightarrow 0$ for $t \gg \gamma^{z/2} \Lambda^{-z}$:

$$a = \frac{(N+2)\hat{u}^*}{2z} \int_0^\infty dx (f_0^K(x, 1) - f_{eq}^K) x^{\frac{2-z}{z}}. \quad (13)$$

The derivation of the free non-equilibrium Keldysh function G_0^K and thus of $f_0^K(x, 1)$ is given in the supplementary section [39]. Once we determine the coefficient a , the exponent θ follows from Eq. (9). For an ohmic bath with $\alpha = 1$, i.e. $z = 2$, we find analytically $a_{z=2} = -\frac{N+2}{N+8} \frac{\epsilon}{4}$, which yields with Eq. (9) the exponent [39]

$$\theta_{z=2} = \frac{N+2}{N+8} \frac{1}{4} \epsilon > 0. \quad (14)$$

For a bath with colored noise, we determine the exponent numerically. Our results for $C_z = \theta \frac{N+8}{N+2} \frac{1}{\epsilon}$ are shown in Fig. 3(a). We find a maximal value for C_z (and thus θ) in the slightly sub-ohmic regime, while $\theta(z \rightarrow 4) \rightarrow 0$ since $\epsilon > 0$ requires at least $z < 4$. For $z < 2$ the exponent decreases and changes sign for $z \approx 1.8$. From Eq. (13) follows that the coefficient a and thus θ can only change sign if equal-time correlations decay non-monotonically. In Fig. 3(b) we show the scaling function $f_0^K(x, 1)$ which proves that this is indeed the case for a super-ohmic bath. Note, in our analysis the limit $z \rightarrow 1$ does not correspond to the closed system with ballistic time evolution as we always consider the limit of bath-dominated dynamics.

For $z = 2$ the value of $C_{z=2}$ turns out to be the same as for a classical phase transition [26, 27]. Identical coefficients for classical and quantum phase transitions might suggest that quantum effects are not important for the quench dynamics. However, considering generic values of z the exponents (for given ϵ) of a classical and quantum

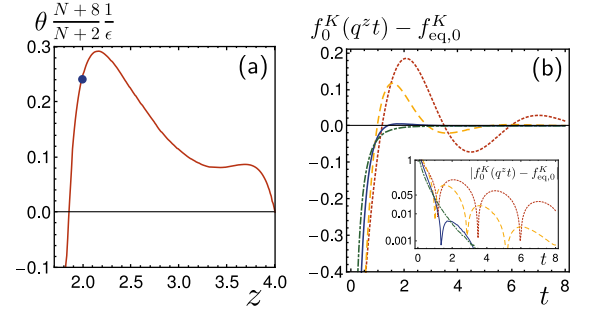


FIG. 3. (Color online) (a) Prethermalization exponent θ as a function of dynamic critical exponent z . Plot shows $C_z = \theta \frac{N+8}{N+2} \frac{1}{\epsilon}$, where N is the number of components of φ and $\epsilon = 4 - d - z$. Blue dot indicates analytical result of Eq. (14). (b) Free Keldysh scaling function $f_0^K(q^z t) - f_{eq,0}^K$ after the quantum quench for different dynamic critical exponents $z = 1.2$ (red), 1.4 (yellow), 2 (blue), 2.5 (green). Inset shows exponential decay of the envelope towards the equilibrium distribution, which becomes algebraic in presence of interactions.

quench are clearly distinct, demonstrating the quantum quench dynamics is in a different universality class as the classical one.

Let us discuss the physical implications of these results: i) *collapse of the correlation length*: We compare the correlation length prior to the quench $\xi_i \propto \delta r_i^{-\nu}$ with its value at the crossover between the prethermalized regime and equilibration $\xi(t^*) \propto \delta r_i^{-\nu/\kappa}$. $\theta > 0$ implies with Eq.(5) that $\kappa > 1$, such that $\xi(t^*) < \xi_i$ for small δr_i . Right after the quench the system falls out of equilibrium and breaks up into many small uncorrelated regions. The correlation length collapses and does not reach its pre-quench value during prethermalization. It takes until after the time scale t^* that the system recovers its initial correlations (see spin configurations in Fig. 1(c)). ii) *order parameter dynamics*: From Eq.(5) follows for $\theta > 0$ that the order parameter grows as function of time. The physical explanation for this behavior follows from our discussion of the path $A \rightarrow C$. $\theta > 0$ leads to a slowing down of the temporal decay of local correlations. On the other hand, the size of correlated regions increases according to the light-cone scale $\xi(t)$. The order parameter grows because of the coarsening that takes place at intermediate time scales where the growth in $\xi(t)$ outweighs the decay of correlations. Thus, the growth of the order parameter $\propto t^\theta$ is caused by the recovery of locally ordered regions after the collapse of the correlation length. The long-time, quasi-adiabatic order-parameter dynamics $\langle \varphi_a(t) \rangle \propto \xi(t)^{-\beta/\nu}$ only sets in when initial correlations are recovered. iii) *equal time correlations*: a straightforward extension of our RG analysis to the scaling function f^K in Eq. (7) yields, instead of the exponential decay shown in Fig. 3(b), a power law decay $f^K(x, 1) = f_{eq}^K - \frac{2\theta}{c_z \sin \frac{\pi}{z}} x^{-2/z}$ with universal co-

efficient proportional to θ . iv) *the regime with $\theta < 0$* : In this case no coarsening growth of the order parameter occurs, yet its decay is slowed down if compared to the quasi-adiabatic regime. In addition, the correlation length recovers before the crossover time t^* is reached. v) *duration of prethermalization*: Since the crossover time t^* diverges for weak quenches, an almost critical system, subject to a sudden change of its parameters, undergoes universal out-of-equilibrium dynamics for arbitrarily long periods of time.

In conclusion, we determined universal behavior that governs quantum critical prethermalization. The intermediate time dynamics of a system that is suddenly moved to a nearby QCP is characterized by a new exponent θ . Owing to the quench, the system instantly falls out of equilibrium and breaks up into small correlated regions. The quantum critical prethermalization describes the recovery after this collapse and extends over long times, depending on the initial distance from the critical point. A quench close to a quantum critical point opens the possibility to quantitatively analyze the universal far-from-equilibrium dynamics of a many body system and to manipulate the crossover between prethermalization and thermalization regimes.

The Young Investigator Group of P.P.O. received financial support from the “Concept for the Future” of the KIT within the framework of the German Excellence Initiative.

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Supplementary Material to “Universal post-quench prethermalization at a quantum critical point”

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BARE RESPONSE AND CORRELATION FUNCTIONS

In order to develop a perturbative renormalization group approach we first need to determine the bare response and correlation functions. In distinction to the equilibrium case, the non-interacting problem far from equilibrium is nontrivial in its own right as we have to determine the bare propagators for our quench-protocol.

For the non-interacting problem it is straightforward to solve the Heisenberg equation of motion of the field operator $\varphi(k, t)$. We find for the Laplace transform

$$\begin{aligned}\varphi(k, \omega) &= \int_0^\infty \varphi(k, t) e^{i(\omega + i0^+)t} dt \\ &= \mathbf{F}(k, \omega) G_{0,\text{eq}}^R(k, \omega),\end{aligned}\quad (1)$$

with force-operator

$$\mathbf{F}(k, \omega) = \mathbf{h}(-k, \omega) + \boldsymbol{\pi}_0(k) - i\omega\boldsymbol{\varphi}_0(k) + \boldsymbol{\xi}(\omega). \quad (2)$$

$\mathbf{h}(k, \omega)$ is a field conjugate to $\boldsymbol{\varphi}(k, \omega)$ that has been added to the Hamiltonian. The $t = 0$ initial values $\boldsymbol{\pi}_0(k)$ and $\boldsymbol{\varphi}_0(k)$ are determined by the pre-quench dynamics. Finally,

$$\boldsymbol{\xi}(t) = - \sum_l \left(c_l \mathbf{X}_{0l} \cos(\Omega_l t) + \frac{c_l \mathbf{P}_{0l}}{\Omega_l} \sin(\Omega_l t) \right) \quad (3)$$

is the operator of the bath-induced force, where $\mathbf{X}_{0l} = \mathbf{X}_l(t = 0)$ and $\mathbf{P}_{0l} = \mathbf{P}_l(t = 0)$ for the oscillator mode l with frequency Ω_l .

$$G_{0,\text{eq}}^R(k, \omega) = \frac{1}{r_0 + k^2 - \omega^2 + \eta(\omega)}. \quad (4)$$

is the bare retarded Green's function without quench, corresponding to equilibrium of the post quench Hamiltonian.

From this solution we now determine the bare retarded and Keldysh Green's functions

$$\begin{aligned}G_0^R(k, t, t') &= -i\theta(t - t') \langle [\varphi_a(k, t), \varphi_a(-k, t')]_- \rangle \\ G_0^K(k, t, t') &= -i \langle [\varphi_a(k, t), \varphi_a(-k, t')]_+ \rangle,\end{aligned}\quad (5)$$

where $[A, B]_\pm = AB \pm BA$. It is convenient to use the double Laplace transform

$$G_0^{R/K}(k, \omega, \omega') = \int_0^\infty dt \int_0^\infty dt' G_0^{R/K}(t, t') e^{i(\omega_+ t + \omega'_+ t')} \quad (6)$$

where we use the notation $\omega_\pm = \omega \pm i0^+$.

For the retarded Green's function we use

$$G_0^R(k, \omega, \omega') = \frac{\delta \langle \varphi_a(k, \omega) \rangle}{\delta h_a(-k, \omega')} \quad (7)$$

and take the expectation value of Eq. (2)

$$\langle \varphi_a(k, \omega) \rangle = h_a(-k, \omega) G_{0,\text{eq}}^R(k, \omega). \quad (8)$$

Using $\delta h_a(k, t) / \delta h_a(k, t') = \delta(t - t')$ it follows

$$\frac{\delta h_a(k, \omega)}{\delta h_a(k, \omega')} = \frac{i}{\omega + \omega' + i0^+}. \quad (9)$$

This yields the result:

$$G_0^R(k, \omega, \omega') = \frac{-iG_{0,\text{eq}}^R(k, \omega)}{\omega + \omega' + i0^+}. \quad (10)$$

Using this result, it follows from simple contour integration that, despite the quench, the bare retarded Green's functions G_0^R on the round trip contour only depends on the time difference $t - t'$ between two events. For the Fourier transform follows the same result as in equilibrium:

$$G_0^R(k, \omega) = G_{0,\text{eq}}^R(k, \omega). \quad (11)$$

The information about the quench is contained in the Keldysh function G_0^K . Inserting the solution of the Heisenberg equation of motion we obtain

$$G_0^K(k, \omega, \omega') = M(k, \omega, \omega') G_0^R(k, \omega) G_0^R(k, \omega'), \quad (12)$$

where we introduced the memory function

$$M(k, \omega, \omega') = -i \langle [F_a(k, \omega), F_a(-k, \omega')]_+ \rangle. \quad (13)$$

In the time domain, Eq. (12) is given as

$$G_0^K(k, t, t') = \int_0^\infty ds ds' M(k, s, s') \times G_0^R(k, t - s) G_0^R(k, t' - s'). \quad (14)$$

This is the out-of-equilibrium version of the fluctuation-dissipation theorem for our quench protocol.

The remaining task is to determine the force-force correlation function that determines $M(k, \omega, \omega')$. Here, the bath induced long-time correlations between pre- and post-quench dynamics, indicated in the Keldysh contour, correspond to mixed terms like $-i \langle [\pi_a(k, \omega), \xi_a(-k, \omega')]_+ \rangle$ in the memory function. Of the numerous terms that emerge many have contributions that diverge in the limit $\omega_c \rightarrow \infty$, with bath cut off ω_c . A straightforward but rather tedious analysis yields that all divergencies cancel and leads to our result for the the memory function:

$$M(k, \omega, \omega') = -i \frac{G_{0,i}^K(k, \omega) + G_{0,i}^K(k, \omega')}{\omega + \omega' + i0^+} \times G_{0,i}^R(k, \omega)^{-1} G_{0,i}^R(k, \omega')^{-1}. \quad (15)$$

Here, the index i refers to the retarded Green's function $G_{0,i}^R(k, \omega)$ and the Laplace transform of the Keldysh function $G_{0,i}^K(k, t - t')$ of a system in equilibrium prior to the quench. Without quench $G_{0,i}^{R(K)} = G_0^{R(K)}$ and one recovers after a few steps that $G_0^K(k, t, t')$ of Eq. (14) takes its equilibrium value, obeying the fluctuation-dissipation theorem. In the classical limit we obtain the results of Ref. 1 for an ohmic bath and Ref. 2 for colored noise.

SCALING LIMIT AND EVALUATION OF THE EXPONENT

From the RG-analysis in the main paper follows for the critical exponent

$$\theta = -\frac{\sin\left(\frac{\pi}{z}\right) u^* (N+2) \Lambda^{-\epsilon} K_d}{\Gamma\left(\frac{2}{z}\right) 2z\gamma^{z/2}} I_0 \quad (16)$$

with

$$I_0 = \frac{q^{4-z}}{\gamma^{\frac{2-z}{2}}} \int_0^\infty dt (iG_0^K(k, t, t) - iG_{0,\text{eq}}^K(k)) t^{\frac{2-z}{z}} \quad (17)$$

and u^* the fixed point value of the interaction. For the Keldysh function we use the result:

$$iG_0^K(k, t, t) = \int_0^\infty ds \int_0^\infty ds' M(s, s', \omega_0^2) G_0^R(k, t - s) G_0^R(k, t - s'), \quad (18)$$

where the argument $\omega_0^2 = \delta r_i + k^2$ indicates that M is obtained from equilibrium Green's functions prior to the quench. δr_i is a measure for the distance from the critical point for $t < 0$. The equilibrium Keldysh function after the quench is

$$iG_{0\text{eq}}^K(k) = \int_0^\infty ds \int_0^\infty ds' M(s, s', k^2) G_0^R(k, t-s) G_0^R(k, t-s'), \quad (19)$$

If we introduce

$$\delta M(s, s') = M(s, s', \omega_0^2) - M(s, s', q^2), \quad (20)$$

it follows

$$I_0 = \frac{q^{4-z}}{\gamma^{\frac{2-z}{2}}} \int_0^\infty ds \int_0^\infty ds' \delta M(s, s') \int_0^\infty dt t^{\frac{2-z}{2}} G_0^R(k, t-s) G_0^R(k, t-s'). \quad (21)$$

Instead of integrations over time, we can easily rewrite this in terms of the frequency dependent memory and retarded Green's functions:

$$I_0 = \frac{-\Gamma(\frac{2}{z})}{i^{2/z}} \frac{q^{4-z}}{\gamma^{\frac{2-z}{2}}} \int_{-\infty}^\infty \frac{d\omega d\omega'}{\pi^2} \frac{\text{Im}G_0^R(k, \omega) \text{Im}G_0^R(k, \omega') \delta M(\omega, \omega')}{(\omega + \omega' - i0^+)^{2/z}}, \quad (22)$$

The evaluation of the integrals is straightforward. For $z = 2$ it can be done analytically and we find

$$I_0(z=2) = -\frac{1}{\pi}$$

of Eq. (17). Performing the limit $z \rightarrow 2$ in the coefficient in Eq. (16) in front of I_0 , we obtain the result for the exponent given in the main paper:

$$\theta(z=2) = \frac{1}{4} \frac{N+2}{N+8} \epsilon. \quad (23)$$

For $z > 2$ the integral Eq. (22) is performed numerically.

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